### MSc Independent Engineering Scholarship (IES) Proposal

**Personal Details**

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| 4 | Award Title | MSc Audio engineering |
| 5 | Proposed Supervisor | Dr. Adam Hill |
| **NOTES:** You can paste material into this form if you wish and expand the sections but the proposal must not exceed 8 pages in length overall, excluding the risk assessment record and the ethics form attached at the end of this proposal. All sections of the risk assessment and ethics form should be completed. | | |

**Dissertation Proposal**

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| 6 | **Proposed Title**  Efficient Acoustic Modelling of Large Spaces Using Time Domain Methods |

**What is the rationale for the proposed IES?**

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| 7 | **Background** Introduction The use of acoustic modelling has expanded from theatre and concert hall design using scale models, through to large format loudspeaker system deployment, environmental noise studies, virtual reality applications, and video game auralization, using innovative tools[1][2]. Many of the packages used to simulate acoustic behaviour, do so using estimation and simplified physical concepts that do not directly simulate wave behaviour or produce direct results.  Time domain methods for solving a simplified wave equation for acoustics may produce direct auralization of sound fields including fundamental acoustic phenomena, as well as providing a visual intuition for acoustic behaviour. These methods are still far from suitable for real-time simulation across the full frequency spectrum, but improvements in method application and parallelisation of processes continues to bring real time acoustic simulation closer to reality[3].  Applying these methods to very large acoustic problems such as arenas, stadia & cathedrals is not trivial, due to in part to the increased scale of computations to undertake. The aim of this dissertation is to explore two methods of solving the acoustic wave equation in a time-stepping fashion, that may significantly reduce total computation time for very large problems. These methods are the Sparse Finite Difference Time Domain Method(SFDTD), and the Pseudospectral Time Domain Method(PSTD). In this proposal, the second order Finite Difference Time Domain Method(FDTD) is introduced, as this is the benchmark method for the dissertation. The two methods of interested are introduced. Finally, some concepts around parallelisation are discussed. Time Domain Numerical Methods Time domain numerical methods for acoustic simulation have some benefits over geometric and frequency domain wave methods. Specifically, time domain numerical methods produce direct and contiguous[[1]](#footnote-1) results across the problem space for the whole of the time being simulated. These methods can also be inclusive of room acoustic behaviour that is not inherent in geometric methods, such as room modes, scattering, sound source interaction, moving sources and receivers [4]. This performance is relatively insensitive to the number of sound sources and receivers in the simulation, unlike geometric and frequency domain wave methods that require problem specific differential equations to be solved.  Time domain numerical methods may be considered like building blocks, in that identical equations are solved multiple times across the domain, and it is possible to solve for varying behaviours without changing a whole model fundamentally. An example of this is the work evaluated by Oxnard *et al*[5], implementing different frequency dependent absorbing boundary condition methods in identical models. The flexibility of time domain numerical methods allows for flexible implementation of moving sound sources, moving receivers, ambisonic sound-field encoding[6], multiple domain modelling[7], heterogenous domain modelling, viscoelastic fluid modelling[8], crossflows, atmospherics. The Finite Difference Time Domain Method However, using time domain numerical methods such as the finite difference time domain (FDTD) method, require a significant number of calculations to be undertaken on large matrices. Applying these methods to very large simulations may not allow for feasible calculation times[2].  The FDTD method was first proposed by Yee[9], and developed as a method for calculating the propagation and material interaction of electromagnetic waves by solving Maxwell’s equations[10]. The method as applied by Botteldooren in his seminal work[11], applies a similar methodology as that used by Yee and the computational electromagnetics fraternity, to acoustic modelling. By applying the same concepts behind the linearized Navier-Stokes model to the solving method used by Yee, it is possible to realise a time stepping solution to the simplified wave equation for fluids[8].  Figure 1Rectiliniarly discretised domain [12]  The explicit second order[[2]](#footnote-2) finite difference time domain method for acoustics is performed as follows:   * The problem space to be simulated is discretised into a rectilinear grid in all dimensions of interest. The grid is conceptually composed of ‘n’ dimensional(nD) matrices of singular points, equally spaced at intervals proportional to half the distance of the smallest wavelength of interest and coefficient of CFL stability. Essentially the discretised problem space (known as the Grid for the rest of the document) is sampled at around 10 points per shortest wavelength of interest. The grid is made up of nD+1 matrices, representing pressure values and velocity potentials across the grid.   Figure 3Illustration of point discretisation of a domain in an acoustic FDTD simulation [13]   * Once other ‘housekeeping’ tasks are completed, constants, sources and other parameters have been prepared, the main calculating loop is entered. The duration of this loop is proportional to the sampling rate, domain size and predominantly the desired acquisition time of the model. The loop executes as follows (assuming the model is 2 dimensional):   + Velocity potentials are calculated in turn across the grid. This is done by differentiating across neighbouring pressure values to the velocity being computed. The pressure gradients are summed and multiplied by a constant relative to the speed of sound, grid resolution and density of the medium. This is then subtracted from the stored velocity potential on the grid at the point being calculated.   + Pressures are calculated in turn across the grid. This is done by carrying out the same process as above. Neighbouring velocity potentials to the pressure point being computed are differentiated, summed, multiplied by the relevant coefficient and then taken from pressure point being calculated.   + These two calculations are undertaken in a continuing leap-frog style and half steps in time and space relative to the proportions of the computational domain. This is an explicit method of solving the wave partial differential equation(PDE), as one variable is calculated by holding all other variables constant for that computation step.   + Finally, a source term is imposed at the source location on the pressure grid, and any results are recorded and displayed. The loop is then continually iterated through until the computation is complete.  Stability, and Significantly Large Matrices This method of solving the acoustic wave equation is simple and flexible, but has a significant drawback. Due to the nature of solving PDEs in this way, there is inherent concern over the stability of the computation. Dispersion error may skew results radically, and sampling theorems such as Nyquist must be satisfied for the simulation to converge correctly. The Courant-Friedrichs-Lewy (CFL) stability condition suggests that for a simulations to converge, the spatial and temporal discretization used must be proportional, and sensitive to the order of the differentiation and so the numerical error[14]. There should be at least 6 if not 10 pressure nodes per the smallest wavelength for the model to converge appropriately. If the highest frequency to be modelled is 20kHz which as a wavelength of 17mm, there must be a node at least every 2.83mm. This is problematic when modelling large spaces, as an arena may be 80m by 120m by 20m and thus have a volume of and would thus require at least 67,844,523 points and ideally 112,941,177 points to be computed stably. With a sampling rate of 48kHz, and assuming the intention of such as simulation were to capture an impulse response over a 10 second period, there would be 54,511,764,705,882 calculations in the computation of the solution alone (almost 651 trillion floating point operations excluding moves). When executing such a computation on a single CPU that is also running an operating system and hosting the integrated development environment that is running the simulation, the final solution may take a significant amount of time to compute (anecdotally on the scale of weeks and months). The Pseudospectral Time Domain Method The Pesudo-spectral time domain (PSTD) and Sparse finite difference time domain (SFDTD) methods may provide a significant increase in calculation speed when compared with a general second-order FDTD implementation[13], [15].  Implementing and optimising PSTD and SFDTD kernels for large simulations may provide a basis on which real time performance could be obtained in further work. The Sparse Finite Difference Time Domain Method The Sparse Finite Difference Time Domain Method is used in electromagnetic simulation, to improve the speed of modelling large and more complex devices (relative to wavelength) such as PIC microcontrollers[16], [17]. Optimisation & Parallelism in MatlabOptimisation Memory Form  Loop optimisation, and relying on previous answers. Parallelism A logical step forward when attempting to solve numerical PDEs quickly is using processor parallelism i.e. implementing the code on several processors or on a processor with multiple cores. A leading example of this can be found in the example material for Mathworks Parallel Computing Toolbox for Matlab. One demonstration provides an example of how parallelising on a graphics card and a CPU are beneficial for differing operations.  Decision based tasks such as IF and Switch statement processing are more ideally suited to CPU parallelism, as a CPU is much faster and optimally tuned for executing these forms of commands when compared to a GPU.  The GPU is more optimal for recurring numeric computation, and thus is more efficient at performing operations such as FFTs and differentiation on large data sets. This is due to the GPU having many cores, and a potentially large pool of local ram to utilise.  The trade of is in passing and returning datasets. This increases the time overhead, as data must be divided, sent, returned and collected.  It is also worth noting that many high-end computer systems available to the public have a CPU and separate GPU. It is less common to be able to purchase computers with multiple CPUs. |

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| 8 | **Aims**   * To implement time domain acoustic modelling methods on very large problems * To determine if these methods present significant improvements in calculations times when compared to a general method |
| 9 | **Objectives**   * To implement a pseudo-spectral time domain method engine for 3D simulations * To implement a locally sparse finite difference time domain method for 3D simulations * To implement a generic second order finite difference time domain method for simulations * To develop a method for indexing large data sets into smaller sets * To benchmark both ‘fast’ methods against the generic method for a simple test problem |
| 10 | **Plan of work**   * Develop a series of Matlab ‘kernel’ functions for FDTD, SFDTD and PSTD simulation * Prove that the results from these kernels is accurate when implemented on a large simulation, by comparing with results of other calculations for the same domain * Develop surrounding code to perform simulations using these kernel * Improve the performance of these kernels with the appropriate code profiling and parallelisation tool * Evaluate performance of the algorithms and suggest areas for improvement, relating to performance speed, with a focus on getting closer to real time simulation |

**Constraints that may restrict the success of the work**

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| 11 | **Foreseeable constraints**  Time  Parallelism of different functions  Access to  Access to equivalent matlab functions in c++ libraries |

**Identifiable risks to the successful completion of the work**

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| 12 | **Foreseeable risks**  The work could fail to improve performance speed  In the long term, the methods could not result in real time performance |

**Resources you envisage utilising to help complete the work**

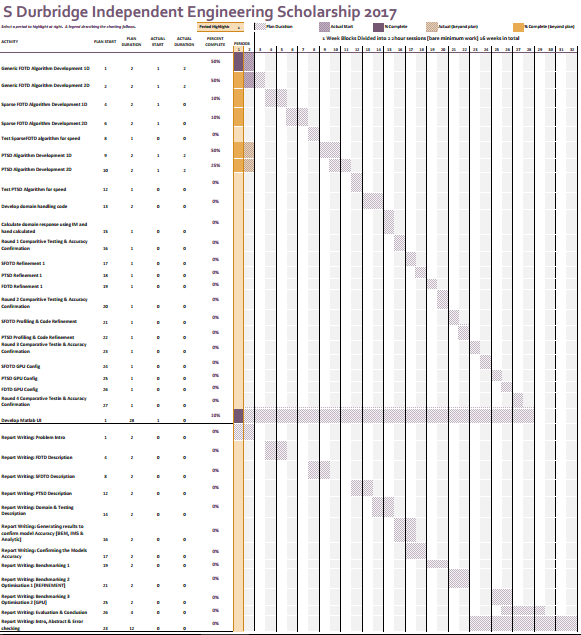
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| 13 | **Textbooks**  Wave and Scattering Methods for Numerical Simulation  S. Bilbao  Wiley, 2004  Master Handbook of Acoustics  Alton, Everest. F  McGraw-Hill, 2009 |
| 14 | **Journals**  Journal of the Acoustical Society of America  <http://asa.scitation.org/journal/jas>  Journal of the Audio Engineering Society  <http://www.aes.org/journal/>  Applied Acoustics  http://www.sciencedirect.com.ezproxy.derby.ac.uk/science/journal/0003682X |
| 15 | **Electronic (internet)**  Library of the Acoustical Society of America  http://asa.scitation.org/  E-Library of the Audio Engineering Society  http://www.aes.org/e-lib/  University of Derby E Library  <http://www.derby.ac.uk/campus/library/>  Digital Audio Effects E Library  http://www.dafx.de/ |
| 16 | **Laboratory equipment and software**  Computer running MATLAB with:  Code Profiler  DSP System Toolbox  Signal Processing Toolbox  Parallel Computing Toolbox  Audio Systems Toolbox |

**Anticipated cost**

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| 17 | **Please enter all costs. Brief Description or explanation.** (£150 maximum) | Cost |
| NVidia GeForce GTX 1060 graphics card – Nvidia graphics card [with large number of CUDA cores](https://developer.nvidia.com/cuda-gpus). [The Matlab Parallel Computing Toolbox has inherent CUDA support](https://uk.mathworks.com/discovery/matlab-gpu.html), allowing for improvement of data processing speed via GPU parallelism with minimal code adaptation. | 150.00 |

**Gantt chart**

Please find an easily seen version of the Gantt chart in the supporting documentation. The chart is split into 16 weeks with 2 sessions per week, to represent the absolute minimum work time required per task (single blocks of between 4 and 8 hours).



Has the IES been agreed with the proposed supervisor? Yes

Explain, if your answer is No……………………………………………………………………………

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| **SUBMISSION:** The completed proposal must be submitted electronically by 11.59 pm on Monday 6th Feb 2017. |

**Record of Risk Assessment**

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| Assessment Reference | | | | | | | | | SDIES\_01 | | | | | | | |
|  | | | | | | | | |  | | | | | | | |  | | |
| Activity assessed | | | | | | | | | | | IES: The Application of Time Domain Acoustical Modelling Methods for Very Large Problems | | | | | | | | | | | | | | | | | | | |
| Persons who may be affected by the activity | | | | | | | | | | | **Simon Durbridge** | | | | | | | | | | | | | | | | | | | |
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| SECTION A : Initial Assessment Overview | | | | | | | | | |
| *Consider the activity or work area and identify if any of the hazards listed below are significant.* | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | Fall of person | | |  | 7 | Machinery | | | | |  | 13 | | Electricity | | | |  | 19 | Substances | | | |  | | 25 | | Drowning | |  |
| 2 | Fall of objects | | |  | 8 | Tools/Equipment | | | | |  | 14 | | Noise or Vibration | | | |  | 20 | High Pressure | | | |  | | 26 | | Psychological effects | |  |
| 3 | Tripping/Slipping | | |  | 9 | Mobile work equipment | | | | |  | 15 | | Hot / Cold Surfaces | | | |  | 21 | Fire/ explosion | | | |  | | 27 | | Human error | |  |
| 4 | Manual handling operations | | |  | 10 | Mechanical lifting equipment | | | | |  | 16 | | Workstation –  layout / space | | | | x | 22 | Lighting | | | |  | | 28 | | Violence | |  |
| 5 | Repetitive work | | | **x** | 11 | Display screen equipment | | | | | x | 17 | | Radiation | | | |  | 23 | Confined space | | | |  | | 29 | | Peripatetic / lone working | |  |
| 6 | Housekeeping / waste material | | |  | 12 | Sharp objects | | | | |  | 18 | | Temperature / weather | | | |  | 24 | Buildings & glazing | | | |  | | 30 | | Other(s) | |  |
|  | | | | | | | | | | | |  | |  | | | |  | | | | | | |
| SECTION B : Second Stage Assessment | | | | | | | | | |  | | | |  | | | | [S = Severity](Risk%20Evaluation%20matrix.doc) | | | | | | |
| *For each hazard identified in Section A complete Section B* [*L = Likelihood*](Risk%20Evaluation%20matrix.doc) | | | | | | | | | | | | | | | | | | | | | | | | |
| Hazard  No. | | Hazard  Description | | | | | | EXISTING CONTROL MEASURES | | | | | | | | | | | | | | S | L | | | | RESIDUAL RISK | |
| **11** | | **Prolonged exposure to computer screen** | | | | | | **Periodic break away from computer** | | | | | | | | | | | | | | **1** | **2** | | | | **Tolerable Risk** | |
| **5** | | **RSI Through keyboard and mouse use** | | | | | | **Periodic break away from computer, with appropriate ancillary/rehab exercises** | | | | | | | | | | | | | | **1** | **2** | | | | **Tolerable Risk** | |
| **16** | | **Damage to computer equipment through excess mess and dust** | | | | | | **Use waste paper/plastic bin, and regularly de-dust computer components** | | | | | | | | | | | | | | **1** | **1** | | | | **Trivial risk** | |
| No. of Section B Continuation sheets used: | | | | | | | | | | | | | | | | | | | | | | | | | | |  | |
| Assessor(s) | | | S. Durbridge, Dr Adam Hill | | | | | | | | | | | | | | Signed | | | | S Durbridge | | | | | | | |
| Date of Assessment | | | | **6/2/2017** | | | Revision No. | | | | | | **1** | | | |

**Request for ethical approval for students on taught programmes**

**Please complete this form and return it to your supervisor as advised in your module handbook. Feedback on your application will be via your supervisor or co-ordinator.**

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| --- | --- | --- | --- |
| **Your Name:** | **Simon Durbride** | | |
| **Student ID:** | **100242305** | | |
| **Unimail address:** | [s.durbridge1@unimail.derby.ac.uk](mailto:s.durbridge1@unimail.derby.ac.uk) | | |
| **Other contact information** |  | | |
| **Programme name and code** | **MSc Audio Engineering (MH6AB)** | | |
| **Module name and code** | **Independent Engineering Scholarship (7EJ998)** | | |
| **Name of supervisor** | **Dr. Adam Hill** | | |
| **Name of co-ordinator** | **Dr. Ahmad Kharaz** | | |
| **Title of proposed research study** | | | |
| **The Application of Time Domain Acoustical Modelling Methods for Very Large Problems** | | | |
| **Supervisor Comments** | | | |
| Are the ethical implications of the proposed research adequately described in this application? | | | Yes ⌧ No ❑ |
| Does the overall study have low, moderate or high risk in terms of ethical implications? | | | Low ⌧ Moderate ❑ High ❑ |
| Does the study method describe a process of research that is ethically sound? | | | Yes ⌧ No ❑ |
|  | | | |
| **Signatures** | | | |
| **The information supplied is, to the best of my knowledge and belief, accurate. I clearly understand my obligations and the rights of the participants. I agree to act at all times in accordance with University of Derby Policy and Code of Practice on Research Ethics:** http://www.derby.ac.uk/research/ethics-and-governance/research-ethics-and-governance | | | |
| **Signature of applicant** | | **Simon Durbridge** | |
| Date of submission by applicant | | 06/02/2017 | |
| **Signature of supervisor** | |  | |
| Date of signature by supervisor | |  | |
| For Committee Use Reference Number (Subject area initials/year/ID number)………………….  Date received……………… Date approved ……………. Signed………………………  Comments | | | |

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| --- |
| **1. What is the aim of your study?**   * To implement time domain acoustic modelling methods to very large problems * To determine if these methods present significant improvements in calculations times when compared to a general method   **What are the objectives for your study?**   * To implement a pseudo-spectral time domain method engine for 2D * To implement a locally sparse finite difference time domain method for 2D * To implement a generic second order finite difference time domain method for 2D * To develop a method for indexing large data sets into smaller sets * To benchmark both ‘fast’ methods against the generic method for a simple test problem |
| **2. Explain the rationale for this study (refer to relevant research literature in your response).**  The use of acoustic modelling has expanded from theatre and concert hall design using scale models, through to large format loudspeaker system deployment, environmental noise studies, virtual reality, and video game auralization using innovative simulation tools[1], [2].  Time domain numerical methods for acoustic simulation have benefits over geometric and frequency domain wave methods. Specifically, time domain numerical methods allow simulations to produce direct and contiguous[[3]](#footnote-3) outputs, while including acoustic behaviour that is not inherent in geometric methods such as room modes[4]. Further, this performance is relatively insensitive to the number of sound sources and receivers in the simulation, unlike geometric and frequency domain wave methods that require separate calculations for multiple parameters across the domain of interest.  However, using time domain numerical methods such as the finite difference time domain (FDTD) method, require doing a significant number of calculations on large sets of data. Applying these methods to very large simulations may not allow for feasible calculation times[2]. The Pesudo-spectral time domain (PSTD) and Sparse finite difference time domain (SFDTD) methods may provide a significant increase in calculation speed when compared with a general second-order FDTD implementation[13], [15].  Implementing and optimising PSTD and SFDTD kernels for large simulations may provide a basis on which real time performance could be obtained in further work. |
| **3. Provide an outline of study design and methods.**   * Develop a series of Matlab ‘kernel’ functions for FDTD, SFDTD and PSTD simulation * Prove that the results from these kernels is accurate when implemented on a large simulation, by comparing with results of other calculations for a reference domain * Develop surrounding code to perform simulations using these kernel * Improve the performance of these kernels with the appropriate code profiling and parallelisation tools * Evaluate performance of the algorithms and suggest areas for improvement, relating to performance speed, with a focus on getting closer to real time simulation |
| **4. Research Ethics**  **Does the proposed study entail ethical considerations No**  **(please delete as appropriate) If you are unsure please seek advice before submitting this form.**  **If ‘No’ provide a statement below to support this position.**  **If ‘Yes’ move on to Question 5.**  **Please note: PROPOSALS INVOLVING HUMAN PARTICIPANTS MUST ADDRESS QUESTIONS 5 - 11.** |
| **5. Please provide a detailed description of the study sample, covering selection, sample profile,   recruitment and if appropriate, inclusion and exclusion criteria.** |
| **6. Are payments or rewards/incentives going to be made to the participants? Yes 🞎 No 🞎   If so, please give details below.** |
| **7. Please indicate how you intend to address each of the following ethical considerations in your study. If you consider that they do not relate to your study please say so.**  **Guidance to completing this section of the form is provided at the end of the document.** |
| **8. Are there any further ethical implications arising from your proposed research? Yes 🞎 No 🞎**  **If your answer was no, please explain why.** |
| **9. Have / do you intend to request ethical approval from any other body/organisation? Yes 🞎 No 🞎**  **If ‘Yes’ – please give details** |
| **10. What resources will you require? (e.g. psychometric scales, IT equipment, specialised software, access to specialist facilities, such as microbiological containment laboratories).** |
| **11. What study materials will you use? (Please give full details here of validated scales, bespoke questionnaires, interview schedules, focus group schedules etc and attach all materials to the application)** |
| **Which of the following have you appended to this application?**   |  |  | | --- | --- | | ❑ Focus group questions | ❑ Psychometric scales | | ❑ Self-completion questionnaire | ❑ Interview questions | | ❑ Other debriefing material | ❑ Covering letter for participants | | ❑ Information sheet about your research study | ❑ Informed consent forms for participants | | ❑ Other (please describe) |  | |

### References

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[17] C. Doerr, “3D Sparse Finite-Difference Time-Domain Simulation of Silicon Photonic Integrated Circuits,” vol. i, pp. 4–6, 2015.

1. Audio samples recorded in time [↑](#footnote-ref-1)
2. Second order in space, first order in time [↑](#footnote-ref-2)
3. Audio samples recorded in time [↑](#footnote-ref-3)